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CHEMICAL PHYSICS OF ATMOSPHERIC PHENOMENA

A Study of Propagation of Nonlinear Acoustic-Gravity Waves in the Middle and Upper Atmosphere by Means of Numerical Modeling*

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Abstract—A numerical model of the vertical propagation and decay of nonlinear acoustic-gravity waves (AGW) from the Earth surface to the upper atmosphere is described. Monochromatic vertical velocity variations at the Earth surface are used as the AGW source in the model. The numerical method for solving three-dimensional hydrodynamic equations is based on finite-difference representation of the fundamental laws of conservation, which makes it possible to calculate not only smooth, but also physically correct generalized solutions of the hydrodynamic equations. The equations are solved in a range of altitudes from the ground up to 500 km. The background temperature, density, molecular viscosity and thermal conductivity coefficient are specified according to standard atmosphere models. The dependence of the characteristics of the waves on the amplitude of the wave source at the lower boundary is examined. The amplitudes of the AGW increase with the altitude, and the waves can break down due to nonlinear effects in the middle and upper atmosphere, depending on the amplitude of the source.

Keywords: middle atmosphere, acoustic-gravity waves, turbulence, nonlinear interactions, numerical modeling, finite-difference method

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INTRODUCTION

Many of the acoustic-gravity waves (AGW) observed in the upper atmosphere originate at tropospheric altitudes [1]. Propagating upwards, AGW can collapse, generating mesoscale disturbances and turbulence in the middle and upper atmosphere. Tropospheric AGW sources may be associated with mesoscale convection and turbulence [1, 2]. Important sources of AGW are tropospheric jet streams at altitudes of 9–12 km [3, 4].

The authors of [5] numerically simulated the propagation and decay of nonlinear AGW in the Venus atmosphere. They studied the propagation of waves in a rectangular region of the atmosphere with the horizontal and vertical dimensions of 120 and 48 km, respectively. The Kelvin–Helmholtz instability and the collapse of internal atmospheric waves were modeled in [6–9]. The authors of these studies investigated the distribution and destruction of waves in rectangular areas with limited dimensions horizontally and vertically. They sought for a solution to the respective hydrodynamic equations in the form of Galerkin expansions, using modifications of the spectral method for transforming the hydrodynamic equations into a system of ordinary differential equations in time for the spectral expansion coefficients.

In the present work, we developed a numerical method for solving three-dimensional atmospheric hydrodynamic equations similar to the two-dimensional method described in [10]. A numerical method is based on finite-difference analogues of the fundamental laws of conservation of energy, mass, and momentum. This approach provides conservativeness of the numerical method and makes it possible to include into consideration nonsmooth solutions to the nonlinear hydrodynamic equations. The fundamental conservation laws enable to construct physically correct generalized solutions to the equations [11, 12].

Here, we briefly describe a three-dimensional algorithm for numerical simulations of nonlinear AGW in the atmosphere. To demonstrate the possibilities of the numerical method and computer code, we calculated the characteristics of waves from a monochromatic source of vertical momentum at the lower boundary. If the wave amplitude is small enough, the solution should correspond to the linear theory of AGW. Therefore, at a small amplitude of the source, waves near it remain harmonic. However, in the vertical propagation of waves, their amplitude increases with

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altitude, and the effect of nonlinearity of the hydrodynamic equations may become significant at large altitudes. In this work, we investigate the dependence of nonlinear effects in the middle and upper atmosphere on the amplitude of the wave at a fixed horizontal phase velocity.

2. SIMULATION ALGORITHM

Let us assume the atmosphere is flat, with the horizontal axes x and y being directed to the east and north, respectively, whereas the axis z is directed upwards. The horizontal components of the velocity are directed along the horizontal axes x and y, while the vertical component w, along the z axis.

A closed system of equations for three-dimensional nonlinear AGW consists of:

(a) the continuity equation

$$\partial\rho/\partial t + \partial(\rho v_\alpha)/\partial x_\alpha = 0, \qquad (1)$$

where ρ is the density, v^α are the velocity components along the coordinate axes x_α, t is the time, summation is implied over repeated Greek indices;

(b) the equations of motion

$$\frac{\partial \rho v_i}{\partial t} + \frac{\partial \rho v_i v_\alpha}{\partial x_\alpha} = -\frac{\partial p}{\partial x_i} - \rho g \delta_{i3} + \rho X_i + \frac{\partial \sigma_{i\alpha}}{\partial x_\alpha}, \; i, \alpha = 1,2,3, \qquad (2)$$

where $\sigma_{i\alpha}$ is the viscous stress tensor, p is the pressure g is the gravitational acceleration, X_i is density of mass forces along the ith axis;

(c) the equation of state of an ideal gas

$$p = \rho RT, \qquad (3)$$

where T is the temperature and R is the universal gas constant for air;

(d) the internal energy equation

$$\rho c_p \frac{dT}{dt} = \frac{dp}{dt} + \frac{dQ}{dt}. \qquad (4)$$

Here, $d/dt = \partial/\partial t + v_\alpha \partial/\partial x_\alpha$, dQ/dt is the rate of heat influx per unit volume. Using (3) and (1) makes it possible to recast (4) as

$$\frac{dp}{dt} = -\gamma p \operatorname{div}\mathbf{v} + (\gamma - 1)\frac{dQ}{dt}, \qquad (5)$$

where $\gamma = c_p/c_v$ is the adiabatic index. The rate of heat influx dQ/dt includes the rate of movement energy dissipation due to molecular viscosity ε_d and thermal conductivity ε_t:

$$\frac{dQ}{dt} = \rho(\varepsilon_t + \varepsilon_d + \varepsilon); \quad \rho\varepsilon_d = \sigma_{\alpha\beta}\frac{\partial v_\alpha}{\partial x_\beta}; \quad \rho\varepsilon_t = -\frac{\partial q_{m\alpha}}{\partial x_\alpha}. \qquad (6)$$

Here, $q_{m\alpha}$ is the molecular heat flux, ε is the specific heat influx other than due to molecular viscosity and thermal conductivity.

Let us calculate the dynamic deviations from the steady-state background values of T_0, ρ_0, and p_0:

$$T' = T - T_0, \quad \rho' = \rho - \rho_0, \quad p' = p - p_0. \qquad (7)$$

At $|T'| \ll T_0$, the wave additives to the molecular viscosity and thermal conductivity coefficient can be disregarded. The values of σ_{ij} and q_{mi} are calculated by the standard formulas:

$$\sigma_{ij} = \mu_0 \Phi_{ij}; \quad \Phi_{ij} = \frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}; \quad q_{mi} = -c_p \text{æ}_0 \frac{\partial T'}{\partial x_i}. \qquad (8)$$

The molecular viscosity and thermal conductivity at moderate temperatures, typical of the middle atmosphere and lower atmosphere are calculated by the Sutherland formula [13]:

$$\mu_0\left[\frac{\text{kg}}{\text{m s}}\right] = \frac{1.46 \cdot 10^{-6}\sqrt{T_0}}{(1 + 110/T_0)}, \quad \text{æ}_0 = \frac{\mu_0}{\Pr_m}, \quad \Pr_m = \frac{4\gamma}{9\gamma - 5}, \qquad (9)$$

where $\Pr_m$ is the Prandtl number for molecular diffusion of heat. At an altitude of 100 km, the molecular viscosity and thermal conductivity coefficients depend on the composition and average molecular weight of the atmosphere. These dependences are described by using various parameterizations (see, e.g., [14]). The dissipation of energy of atmospheric motions is boosted by irregular turbulent disturbances [15]. The evolution of turbulent disturbances with dimensions larger than the spatial grid steps is described by the nonlinear model under consideration.

Subgrid-scale turbulence can be taken into account in the first approximation by replacing μ_0 and æ$_0$ in (7) by

$$\mu_t = \mu_0 + \rho_0 K, \quad \text{æ}_t = \text{æ}_0 + \rho_0 K_t, \quad K_t = K/\Pr_t, \qquad (10)$$

where $\Pr_t$ is the turbulent Prandtl number, and K and K_t are the kinematic eddy viscosity and thermal conductivity, respectively.

At the height of h = 500 km, the upper boundary conditions are specified in the standard form:

$$\left(\frac{\partial T}{\partial z}\right)_{z=h} = 0, \quad \left(\frac{\partial u}{\partial z}\right)_{z=h} = 0, \quad \left(\frac{\partial v}{\partial z}\right)_{z=h} = 0, \quad (w)_{z=h} = 0. \qquad (11)$$

Some of the boundary conditions at the Earth surface were found to have only slight influence on the wave processes outside the boundary layer near the surface, but they are necessary from the mathematical point of view. These are the following conditions:

$$(T')_{z=0} = 0, \quad (u)_{z=0} = 0, \quad (v)_{z=0} = 0. \qquad (12)$$