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# Peculiarities of Approximation of Meteorological Characteristics Having Vertical Gradient Discontinuities Close to the Tropopause

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**Abstract**—Carried out is an analysis of errors of different approximation methods for separating the long- and short-wave parts of the spectra of the vertical distribution of meteorological variables and gas components in the areas of dramatic variations of their vertical gradients close to the tropopause. Proposed is an integrated approach using the polynomial, piecewise-linear, and spectral filtration and enabling to minimize the errors of approximation of long- and short-wave components of profiles of meteorological characteristics in the troposphere and stratosphere including the tropopause area. The method is useful for analyzing the mean values and mesoscale variations of meteorological parameters measured by the modern balloon and satellite systems.

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## 1. INTRODUCTION

The dramatic changes in lapse rates are possible at the level of the tropopause located at the height of 10–20 km in the atmosphere depending on the latitude and the season [8]. Such variations can hamper the approximation of the vertical distribution of temperature and other meteorological characteristics by smooth mathematical functions in the tropopause area.

In recent years, significant attention has been paid to the study of mesoscale dynamic processes being of great importance for the dynamics of the lower and middle atmosphere [7]. To analyze the atmospheric processes, the satellite remote high-resolution measurements are used ensuring the reception of the global data on the dynamic structures, inhomogeneities of planetary scale, and mesoscale disturbances of meteorological parameters. The authors of [10] used the satellite data of LIMS (Limb Infrared Monitor of the Stratosphere) instrument; the authors of [9] presented the results of CRISTA (Cryogenic Infrared Spectrometers and Telescopes for the Atmosphere) experiment satellite data processing; the authors of [13, 21] studied the global fields of temperature and their mesoscale disturbances in the stratosphere as derived from the data of MLS (Microwave Limb Sounder) instrument installed on the UARS satellite.

The GPS (Global Positioning System) satellites were used for studying the temperature, density, and the content of water vapor in the atmosphere at the height up to 30–35 km above the Earth surface [2, 11, 14, 16, 19]. The study of mesoscale structures continued using the data of CHAMP low-orbit satellite launched in April 2001 [1, 15] and COSMIC satellites launched in 2006 [4–6, 17, 18]. As a rule, in order to carry such studies out, numerical filtration is used for smoothing the vertical distributions of atmospheric refraction index and temperature as well as for the determination of the intensity of mesoscale disturbances. However, such filtration can cause errors and overestimate the amplitudes of disturbances at the altitudes, where the dramatic changes take place in the vertical gradients of meteorological characteristics close to the tropopause. To reduce the phase errors during the analysis of vertical and horizontal quasiwave structures, the special methods were worked out [12, 18] using the joint measurements of the whole group of COSMIC satellites. However, these methods cannot be used for analyzing the data from individual satellites similar to the CHAMP satellite (till 2006). In this case, it is proposed to use the simplified methods of data filtration for smoothing the vertical distributions [3, 15]. In spite of quite extensive studies, the problems of approximation and filtration of vertical distributions of temperature and of other parameters having dramatic variations of vertical gradients in the tropopause area have not been solved so far.

In the present paper, the errors are analyzed of typical approximations of meteorological variables by the smooth mathematical functions that arise in case of their use in the vicinity of vertical derivative discontinuities. An integrated approach is proposed using the polynomial, piecewise-linear, and spectral filters for reducing the errors of approximation of profiles of meteorological variables in the wide range of altitudes in the troposphere and stratosphere including the tropopause area.

## 2. METHODS OF SMOOTHING PROFILES OF METEOROLOGICAL CHARACTERISTICS

When studying the variations of atmospheric characteristics, the values of $y(z)$ depending on altitude are often presented in the following form:

$$y(z) = \tilde{y}(z) + \delta y(z), \tag{1}$$

where $\tilde{y}(z)$ is the smoothed value caused by the long-wave part of the spectrum of atmospheric variations; $\delta y(z)$ is the short-wave deviations from the smoothed values. The low-frequency numerical filters based on different methods of approximation of vertical distributions of analyzed parameters are often used for computing the smoothed values. In the present research, three types of standard approximating functions are considered. The first type is the polynomial approximation

$$\tilde{y}(z) = \sum_{k=0}^{M} a_k z^k, \tag{2}$$

where $a_k$ are the coefficients held by the $k$th powers of independent variable. The second type is the approximation of the long-wave component by the sum $P$ of spectral harmonics, the spectral approximation:

$$\tilde{y}(z) = A_0 + \sum_{i=1}^{P} (A_i \cos m_i z + B_i \sin m_i z), \tag{3}$$

where $m_i = 2\pi i/\Delta z$ is the vertical wave number; $A_i$ and $B_i$ are the amplitudes; $\Delta z$ is the altitude range used for the approximation. Close to the tropopause, the approximation of vertical distributions by the piecewise-linear function referred to the third type can be used:

$$\tilde{y}(z) = \begin{cases} a + b(z - z_0) & \text{for} \quad z < z_0; \\ a + c(z - z_0) & \text{for} \quad z > z_0, \end{cases} \tag{4}$$

where $z_0$ is the height of tropopause; $a$, $b$, and $c$ are the constants. The values of coefficients in functions (2)–(4) were determined for each altitude $z$ by means of approximation using the least-squares method for the moving intervals of the altitude $\Delta z$. The experimental values $y_i = y(z_i)$ used for the approximation can be unequally spaced along the vertical. Polynomials (2) are used with $M = 1$ (linear filter) and $M = 2$ (square filter). In function (3), $P = 1$ (spectral filter) is specified. For the piecewise-linear function (4), the search of the value $z_0$ is carried out; $z_0$ ensures the minimum sum of squared deviations within each interval $\Delta z$.

## 3. RESULTS OF FILTRATION

In Fig. 1, the results are presented of using the filters (2)–(4) for approximating the artificial profile of temperature that simulates the long- and short-wave parts of the spectrum of atmospheric variations. It is formed by two linear functions with the lapse rates of –6 and 3 K/km below and above the level $z = 15$ km, respectively, and by the sinusoid with the constant amplitude of 3 K with the vertical wave length $\lambda_z = 5$ km. The approximation of the model profile by the functions (2)–(4) is carried out by the least-squares method using the moving intervals for the height $\Delta z = 10$ km. In Fig. 1b, much higher amplitudes (than the specified one) of the short-wave component of the spectrum can be seen. They are induced by the significant divergence of the specified and the smoothed profiles in Fig. 1a close to the tropopause (at the height $z = 15$ km) associated with the errors of square and spectral approximations. The piecewise-linear approximation does not cause any apparent changes in the amplitude of short-wave sinusoidal disturbance in the tropopause area (Fig. 1b). Outside the altitude range of ±5 km from the tropopause, all three approximations give similar short-wave sinusoids (Fig. 1b) with slightly differing amplitudes. In the 5-km vicinity of the lower and upper boundaries of the analyzed height range, the amplitudes of the short-wave sinusoidal temperature disturbance (Fig. 1b) are lower as compared with the specified value of 3 K, and such underestimation of amplitudes is larger for the spectral approximation.

Differences in the amplitude of short-wave sinusoidal disturbances presented in Fig. 1b for the approximation at the altitudes situated at the distance of more than 5 km from the tropopause are caused by the differences in smoothing effects of numerical filters under consideration which can reduce the ratio $r = \theta_a/\theta$,