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Numerical Modeling of the Propagation of Nonlinear Acoustic-Gravity Waves in the Middle and Upper Atmosphere

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Abstract—A numerical algorithm for modeling the vertical propagation and breaking of nonlinear acoustic-gravity waves (AGWs) from the Earth's surface to the upper atmosphere is described in brief. Monochromatic variations in the vertical velocity at the Earth's surface are used as an AGW source in the model. The algorithm for solving atmospheric hydrodynamic equations is based on three-dimensional finite-difference analogues of fundamental conservation laws. This approach selects physically correct generalized solutions to hydrodynamic equations. A numerical simulation is carried out in an altitude region from the Earth's surface to 500 km. Vertical profiles of the background temperature, density, and coefficients of molecular viscosity and heat conduction are taken from the standard atmosphere models. Calculations are made for different amplitudes of lower-boundary wave forcing. The AGW amplitudes increase with altitude, and waves may break in the middle and upper atmosphere.

Keywords: middle atmosphere, acoustic-gravity waves, turbulence, nonlinear interactions, numerical modeling, finite-difference method

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1. INTRODUCTION

Acoustic-gravity waves (AGWs) observed in the upper atmosphere may be generated at tropospheric heights [1]. Propagating upward, AGWs may lose stability, thereby triggering mesoscale disturbances and turbulence in the middle and upper atmosphere. Some AGW sources may be produced by mesoscale convection and turbulence in the troposphere [1, 2]. These sources may be most active at 9- to 12-km altitudes of tropospheric jet streams [3, 4].

The propagation and breaking of nonlinear AGWs in the atmosphere of Venus was modeled numerically in [5]. The authors simulated wave propagation in a rectangular atmospheric region with horizontal and vertical sizes of 120 and 48 km, respectively. Others [6–9] simulated the breaking of internal gravity waves and the Kelvin–Helmholtz instability. Their models are three-dimensional, and wave propagation and breaking was simulated in a rectangular domain with limited horizontal and vertical sizes. In [6–9], the authors searched for solutions to hydrodynamic equations in the form of Galerkin-type decompositions and used modifications of the spectral method to turn partial differential hydrodynamic equations into ordinary differential equations over time for spectral decomposition coefficients.

In the present study, for solving atmospheric hydrodynamic equations, we have developed a three-dimensional numerical method similar to a two-dimensional algorithm using finite-difference analogues of the fundamental conservation laws for energy, mass, and momentum described in [10]. An approach like this keeps the numerical method conservative and allows an examination of nonsmooth solutions to nonlinear dynamic equations. This enables us to select physically correct generalized solutions to equations [11, 12].

The present paper briefly describes a three-dimensional algorithm for the numerical modeling of nonlinear AGWs in the atmosphere. To illustrate how the algorithm works, calculations are made for waves generated by monochromatic variations of vertical velocity at the lower boundary. If the amplitude of the source is small enough, the solution can be expected to fit the linear theory of AGWs. Thus, the waves near the source may remain harmonic. However, as the amplitude increases with altitude, the influence of the nonlinearity of hydrodynamic equations may become significant at high altitudes. Such a dependence of the nonlinear effects in the middle and upper atmosphere on the wave amplitude at a fixed horizontal phase speed is illustrated in this paper.

2. ALGORITHM OF NUMERICAL MODELING

Our three-dimensional numerical model of nonlinear AGWs in a plane atmosphere calculates the horizontal u and v and vertical w velocity components along the horizontal x and y axes directed eastward and northward and along the vertical z axis, respectively. The set of solvable three-dimensional nonlinear equations of the hydrodynamic model includes

(a) the equation of continuity

$$\partial\rho/\partial t + \partial(\rho v_\alpha)/\partial x_\alpha = 0, \quad (1)$$

where t is time, ρ is density, v_α denotes velocity components along the coordinate x_α axes, and repeated Greek subscripts imply summation;

(b) the equations of motion

$$\frac{\partial\rho v_i}{\partial t} + \frac{\partial\rho v_i v_\alpha}{\partial x_\alpha} = -\frac{\partial p}{\partial x_i} - \rho g\delta_{i3} + \rho X_i + \frac{\partial\sigma_{i\alpha}}{\partial x_\alpha}, \quad (2)$$
$$i, \alpha = 1, 2, 3$$

where p is pressure, g is the gravitational acceleration, X_i is the density of mass forces along the ith axis, and $\sigma_{i\alpha}$ is the viscous stress tensor;

(c) the equation of state of an ideal gas

$$p = \rho RT, \quad (3)$$

where T is the temperature and R is the gas constant for air;

(d) the energy conservation equation in the form of the heat influx equation

$$\rho c_p \frac{dT}{dt} = \frac{dp}{dt} + \frac{dQ}{dt}, \quad (4)$$

where $d/dt = \partial/\partial t + v_\alpha\partial/\partial x_\alpha$, dQ/dt is the rate of heat influx per unit volume. Using (3) and (1), we can write (4) as

$$\frac{dp}{dt} = -\gamma p div \mathbf{v} + (\gamma - 1)\frac{dQ}{dt}, \quad (5)$$

where $\gamma = c_p/c_v$ is an adiabatic index. The heat influx rate dQ/dt contains the rates of energy dissipation of motions due to molecular viscosity, ε_d, and thermal conduction, ε_t:

$$\frac{dQ}{dt} = \rho(\varepsilon_t + \varepsilon_d + \varepsilon); \quad \rho\varepsilon_d = \sigma_{\alpha\beta}\frac{\partial v_\alpha}{\partial x_\beta}; \quad \rho\varepsilon_t = -\frac{\partial q_{m\alpha}}{\partial x_\alpha}, (6)$$

where $q_{m\alpha}$ is the molecular heat flux and ε is the specific heat influx different from molecular viscosity and thermal conduction. Equations of motion (2) and heat influx equations (5) and (6) contain viscous stress tensor σ_{ij} and molecular heat flux q_{mi}. After the numerical integration of Eqs. (1)–(5), dynamic deviations from the stationary background values T_0, ρ_0, and p_0 are calculated:

$$T' = T - T_0, \quad \rho' = \rho - \rho_0, \quad p' = p - p_0. \quad (7)$$

If the amplitudes of wave temperature variations $|T'| \ll T_0$, the wave components of the coefficients of molecular viscosity and heat conduction can be neglected. Then for calculating σ_{ij} and q_{mi}, we can use standard formulas

$$\sigma_{ij} = \mu_0\Phi_{ij}; \quad \Phi_{ij} = \frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}; \quad q_{mi} = -c_p\kappa_0\frac{\partial T}{\partial x_i}. \quad (8)$$

To calculate the coefficients of molecular viscosity and heat conduction at moderate temperatures characteristic of the middle atmosphere and lower thermosphere, Sutherland's formula [13] can be used

$$\mu_0 = \frac{1.46 \times 10^{-6}\sqrt{T_0}}{(1 + 110/T_0)}\left(\frac{kg}{m \times s}\right);$$
$$\kappa_0 = \frac{\mu_0}{\mathrm{Pr}_m}; \quad \mathrm{Pr}_m = \frac{4\gamma}{9\gamma - 5}, \quad (9)$$

where Pr_m is the Prandtl number for the molecular heat diffusion. Above 100 km, the coefficients of molecular viscosity and heat conduction become dependent on the composition and mean molecular weight of the atmosphere. Parameterizations of different complexity described in the literature (see [14]) can be used here. The energy dissipation of atmospheric motions is also influenced by irregular turbulent disturbances [15]. The evolution of such disturbances with sizes larger than spatial grid spacing is described by the nonlinear model itself. The influence of subgrid-scale turbulence at a first approximation can be taken into account by replacing μ_0 and κ_0 in (7) by

$$\mu_t = \mu_0 + \rho_0 K; \quad \kappa_t = \kappa_0 + \rho_0 K_t; \quad K_t = K/\mathrm{Pr}_t, \quad (10)$$

where K and K_t are the kinematic coefficients of turbulent viscosity and thermal conduction, respectively, and Pr_t is the turbulent Prandtl number.

The upper boundary conditions at $h = 500$ km are written as

$$\left(\frac{\partial T}{\partial z}\right)_{z=h} = 0, \quad \left(\frac{\partial u}{\partial z}\right)_{z=h} = 0,$$
$$\left(\frac{\partial v}{\partial z}\right)_{z=h} = 0, \quad (w)_{z=h} = 0. \quad (11)$$

The lower boundary conditions at the Earth's surface are of the following form:

$$(T')_{z=0} = 0, \quad (u)_{z=0} = 0, \quad (v)_{z=0} = 0. \quad (12)$$

Variations in the vertical velocity at the lower boundary may be a source of waves in the model considered here. In our research they are given in the form of a plane wave

$$(w)_{z=0} = W_0\cos(\sigma t - kx - ly), \quad (13)$$

where W_0 is the amplitude, σ is the frequency, and k and l are the wave numbers along the horizontal x and y axes, respectively. Such plane waves may represent spectral components of tropospheric convective, turbulent, and orographic AGW sources, the action of which can be described by variations of effective verti-